

Competition in the Hierarchical Platform Markets

Minji Kang*and Wiroy Shin†

January 31, 2022

Working draft: Please do not cite, circulate or quote without
permission of the authors.

Abstract

To understand market equilibria in the platform economy, it is important to consider competition at every level. In the recent literature on platform competition, Karle et al.(2020) propose a platform competition model accompanied by seller competition in each platform and show how the platform market structure is driven by the product market competition. In this paper, we extend the model of Karle et al.(2020) and rationalize a process that a platform acquires a dominant position in the market. We specify diverse platform market environments in the model, such as multi-homing choices and tastes of consumers, tying, and advertisements on platforms, and analyze how the tipping equilibria and welfare concentration evolve.

1 Introduction

The influence of large platforms on industries and markets is becoming stronger, and the importance of competition policies to regulate their influence (market

*KIET, minjikang@kiet.re.kr

†KIET, wiroypsu@gmail.com

◊ An insightful suggestion by Yangshin Park led to the positive results presented in this paper.

dominance) is increasing. Due to the nature of two-sided markets where network effects of consumers and sellers are crucial, a small number of platforms have market dominance in the market. Such a platform market's concentration is not solely determined by platform themselves: it is also affected by competition at another level, that is, competition of sellers(or platforms who provide services within a platform) on the platform and consumers' strategic choices. This study investigates such strategic relations between agents' choices in the platform economy and explain processes of tipping in various platform market conditions.

Armstrong(2006), Caillaud and Jullien(2003), and Rochet and Tirole(2003, 2006) are seminal studies of competition in two-sided markets. These early studies focus on platform entry choices, network effects among platform participants, and the corresponding tipping equilibrium. In detail, these studies derive an optimal registration fee strategy imposed by the platform on the user groups, when the indirect network effect exists. However, these studies do not consider competition at the seller-side level and transactions between sellers and consumers. Dudey (1990) and Ellison and Fudenberg (2003) study sellers' competition in the product market and the subsequent strategic platform selection of sellers. These studies imply a situation in which sellers monopolize profits through market dominance as the platform does not charge a registration fee to sellers, but platform's market dominance is not explained. Developing the aforementioned studies, a recent study by Karle et al. (2020) propose a platform competition model accompanied by seller competition and show how the platform market structure is driven by the product market competition.

In this paper, we extend the model of Karle et al.(2020) incorporating various market conditions and rationalize a process through which a platform acquires a dominant position. In particular, while the model of Karle et al.(2020) consider

sellers' differentiation, our study simplifies the discussion by assuming homogeneous good market with Bertrand competition. And then, we specify diverse platform market environments in the model, such as multi-homing choices and tastes of consumers, tying, and advertisements on platforms. The advantage of our simple model is that it can explore the various actual platform market issues, and results of the analysis are highly intuitive, leading to discussions in welfare and policy implications.

The structure of this paper is as follows. Section 2 presents a baseline model where only sellers' multihoming is allowed. We specify best responses of agents in every stage of the game and derive an equilibrium where tipping does not occur. In section 3, we analyze the platform market competition in various situations and show how the tipping occur. In section 4, we discuss policy implications and concludes the paper.

2 The Baseline Model

2.1 Environment

Players In a platform market, three economic agents group exist: platforms, sellers, and consumers. For simplicity, we assume that there are two platforms and two sellers, and a continuum of consumers.

- platforms: $l \in \{A, B\} = L$
- sellers: $i \in \{1, 2\} = I$
- consumers: $\theta \in [0, 1] = \Theta$

Strategies In our baseline model, players in the same group are homogeneous: platforms provide the same level of service quality, and sellers sell a homogeneous good (or service). While multihoming of sellers is possible, multihoming

for consumers is not allowed. The following timeline captures a transaction procedure in the platform market.

1. Platform l set a listing fee f^l : $f^l \in [0, \infty)$.
2. After observing the listing fees, seller i chooses a platform(or multihoming) to list his product/service: $s_i \in \{A, B, MH\}$ (MH: multihoming).
3. Based on the platform-seller composition, consumer θ chooses a platform to join: $s_\theta \in L$.
4. Seller i sets price p_i^l of his product: $p_i^l \in [0, \infty)$.
5. After observing prices of sellers on platform, consumer θ purchases an object from seller i : $s_\theta^l \in I$.

Payoffs Consumers' valuation on the object is fixed, and a direct network effect is determined by a measure of consumers on a platform. Notations related to players' payoffs are as follows:

- x : value of an object
- η^l : size of consumers on platform l
- γ : coefficient of the network effect in consumers' utility
- μ_i^l : size of consumers who purchased an object from seller i on platform l
- n^l : the number of sellers on platform l

The cost of platform services and the cost of sellers excluding the listing fee are normalized to 0. The profit of platforms is generated by the listing fee. The profit of sellers is a net value of total revenue and the listing fee. The utility

of consumers is determined by the purchasing decision and the network effect. The players' utility functions are as follows.

- $u_l(f^l; s_i, p_i^l, s_\theta, s_\theta^l) = n^l \cdot f^l$
- $u_i(s_i, p_i^l; f^l, s_\theta, s_\theta^l) = \begin{cases} \mu_i^l \cdot p_i^l - f^l = i, & s_i \in L \\ \sum_{s_i} \mu_i^l \cdot p_i^l - f^l, & s_i = MH \end{cases}$
- $u_\theta(s_\theta, s_\theta^l; f^l, s_i, p_i^l) = x - p_i^l + \gamma \cdot \eta^l$

We assume that outside options for sellers and consumers exist: sellers may not list their products on platforms, and consumers may purchase nothing. The utility level of the agent to choose the outside option is assumed to be zero.

2.2 The hierarchical platform competition game

Our solution concept is subgame perfect Nash equilibrium, and we use backward induction to derive the equilibrium. In this section, we analyze best responses of the players in the each stage of the game.

Note that since sellers are symmetric, the identity of sellers do not matter for platforms and consumers. Therefore, only the number of sellers is crucial for other players to decide their strategies. Table 1 describes a possible market structure of sellers in each platform after sellers' finalizing their platform choices: cases of monopoly by seller i , duopoly, no sellers are denoted by $m(i)$, d , \emptyset , respectively.

If both sellers 1 and 2 choose platform A, A's market structure becomes an oligopoly (A: d), and no sellers market is formed in B (B: \emptyset). When seller 1 chooses A and 2 chooses B, a monopoly by seller 1 is formed in A (A: $m(1)$), and a monopoly by seller 2 is formed in B (B: $m(2)$). When Seller 1 decides

$s_1 \backslash s_2$	A	B	MH
A	A: d B: \emptyset	A: m(1) B: m(2)	A: d B: m(2)
B	A: m(2) B: m(1)	A: \emptyset B: d	A: m(2) B: d
MH	A: d B: m(1)	A: m(1) B: d	A: d B: d

Table 1: Product market structures in each platform

multihoming and seller 2 chooses A, an oligopoly is formed on platform A (A: d), and a monopoly market by seller 1 is formed on B (B: m(1))).

Based on this seller-side (product) market structure in each platform, sellers face a price competition, and choose either monopoly price $p_i^l(m)$, or duopoly price $p_i^l(d)$.

2.2.1 Step 5: consumers' seller selections

In step 5, when a consumer exists on a specific platform, a seller is selected by the product prices on the platform and the network effect.

Lemma 1. *Consumer θ in platform l face one of the three product market structures: monopoly, duopoly, no sellers. In each case, consumer θ selects a seller using the following standards.*

1. *Case m(i)*

purchasing from i

$$\Leftrightarrow u_\theta(s_\theta^l = i; \cdot) \geq 0$$

$$\Leftrightarrow p_i^l(m) \leq x + \gamma \cdot \eta^l$$

2. *Case d*

purchasing from i

$$\Leftrightarrow u_{\theta}(s_{\theta}^l = i; \cdot) \geq u_{\theta}(s_{\theta}^l = j; \cdot)$$

$$\Leftrightarrow p_i^l(d) \leq p_j^l(d)$$

3. *Case \emptyset*

no purchasing

$$u_{\theta} = 0$$

Proof. In case of m(i), consumers purchase from the monopoly seller if consumption guarantee consumers' outside option utility 0. In case of d, consumers compare the prices, and if there is no seller, consumers do not purchase. \square

2.2.2 Step 4: Sellers' price setting

In each market structure (see Table 1), sellers perform a price competition.

Lemma 2. *Sellers set their prices based on the market structure as follows.*

1. *platform l: m(i), platform k: m(j)*

$$p_i^l(m) = x + \gamma \cdot \eta^l$$

$$p_j^k(m) = x + \gamma \cdot \eta^k$$

2. *platform l: d, platform k: \emptyset*

$$p_i^l(d) = p_j^l(d) = 0$$

3. *platform l: m(i), platform k: d*

$$p_i^l(m) = p_j^l(m) = x + \gamma \cdot \eta^l$$

$$p_i^k(d) = p_j^k(d) = 0$$

4. platform l : d , platform k : d

$$p_i^l(d) = p_j^l(d) = p_i^k(d) = p_j^k(d) = 0$$

Proof. (a) In case of $m(\cdot)$, sellers can set the price s.t. $u_\theta = 0$.

(b) In case of d , the Bertrand competition results in 0 price (a production cost).

Note that the listing fee is a sunk cost for sellers in this stage. \square

2.2.3 Step 3: Consumers' platform selections

After observing sellers' platform choices in Step 2, in Step 3 consumers select platforms. Based on the observed market structure, consumers can infer sellers' price setting (the result of Step 4), and thus can predict their final utility levels. The following lemma shows that consumers preference on duopoly market.

Lemma 3. *Consumers select a platform according to the product market structure determined in step 2 as follows:*

1. *If all platforms form a monopoly market at the sellers level, all consumers choose one platform A (or B), or choose A or B randomly.*
2. *If sellers exist on one specific platform only, consumers choose the platform where sellers exist.*
3. *If a specific seller forms a monopoly market on one platform and a duopoly market is formed on the other platform, consumers choose the platform where the duopoly market is formed.*
4. *If all platforms form duopoly at the sellers level, all consumers choose one platform A (or B) equally, or choose A or B with a probability of 0.5.*

Proof. Based on Lemma 1 and 2, consumer utility levels depending on platform selections can be computed as follows.

1. platform l : $m(i)$, platform k : $m(j)$

$$u_\theta(s_\theta = l; \cdot) = x + \gamma \cdot \eta^l - p_i^l(m) = 0$$

$$u_\theta(s_\theta = k; \cdot) = x + \gamma \cdot \eta^k - p_j^k(m) = 0$$

\Rightarrow Consumers are indifferent between the two platforms.

2. platform l : d , platform k : \emptyset

$$u_\theta(s_\theta = l; \cdot) = x + \gamma \cdot \eta^l - p_i^l(d) = x + \gamma \cdot \eta^l$$

$$u_\theta(s_\theta = k; \cdot) = 0$$

\Rightarrow Consumers prefer a platform where a duopoly sellers market exist.

3. platform l : $m(i)$, platform k : d

$$u_\theta(s_\theta = l; \cdot) = x + \gamma \cdot \eta^l - p_i^l(m) = 0$$

$$u_\theta(s_\theta = k; \cdot) = x + \gamma \cdot \eta^k - p_i^l(d) = x + \gamma \cdot \eta^k$$

\Rightarrow Consumers prefer a platform where a duopoly sellers market exist.

4. platform l : d , platform k : d

$$u_\theta(s_\theta = l; \cdot) = x + \gamma \cdot \eta^l - p_i^l(d) = x + \gamma \cdot \eta^l$$

$$u_\theta(s_\theta = k; \cdot) = x + \gamma \cdot \eta^k - p_i^l(d) = x + \gamma \cdot \eta^k$$

\Rightarrow Consumers prefer a platform where the size of consumers is larger. \square

2.2.4 Step 2: Sellers' platform selections

Let y be a vector representing the product market structure on platform A and B . For example, if seller 1, 2 forms monopoly on platform A and B respectively, $y = (m(1), m(2))$. Following Lemma 3, we can define a correspondence of y ,

$\eta^l(y)$, presenting the size of consumers in each platform.

The following Table 2 and Table 3 show the profit of sellers based on Lemma 1 and 2, and Lemma 3 respectively. Note that the market structure is determined by Table 1. For example, if $y = (m(1), m(2))$, $u_1(s_1 = A; s_2 = B, \cdot) = -f^A$, $u_2(s_2 = B; s_1 = A, \cdot) = -f^B$.

Table 2: The profit of sellers based on Lemma 1 and 2

$s_1 \setminus s_2$	A	B	MH
A	$-f^A$ $-f^A$	$\eta^A(y) \cdot (x + \gamma \cdot \eta^A(y)) - f^A$ $\eta^B(y) \cdot (x + \gamma \cdot \eta^B(y)) - f^B$	$-f^A$ $\eta^B(y) \cdot (x + \gamma \cdot \eta^B(y)) - (f^A + f^B)$
B	$\eta^B(y) \cdot (x + \gamma \cdot \eta^B(y)) - f^B$ $\eta^A(y) \cdot (x + \gamma \cdot \eta^A(y)) - f^A$	$-f^B$ $-f^B$	$-f^B$ $\eta^A(y) \cdot (x + \gamma \cdot \eta^A(y)) - (f^A + f^B)$
MH	$\eta^B(y) \cdot (x + \gamma \cdot \eta^B(y)) - (f^A + f^B)$ $-f^A$	$\eta^A(y) \cdot (x + \gamma \cdot \eta^A(y)) - (f^A + f^B)$ $-f^B$	$-(f^A + f^B)$ $-(f^A + f^B)$

11

Table 3: The profit of sellers based on Lemma 3

$s_1 \setminus s_2$	A	B	MH
A	$-f^A$ $-f^A$	$\eta^A(y) \cdot (x + \gamma \cdot \eta^A(y)) - f^A$ $\eta^B(y) \cdot (x + \gamma \cdot \eta^B(y)) - f^B$	$-f^A$ $-(f^A + f^B)$
B	$\eta^B(y) \cdot (x + \gamma \cdot \eta^B(y)) - f^B$ $\eta^A(y) \cdot (x + \gamma \cdot \eta^A(y)) - f^A$	$-f^B$ $-f^B$	$-f^B$ $-(f^A + f^B)$
MH	$-(f^A + f^B)$ $-f^A$	$-(f^A + f^B)$ $-f^B$	$-(f^A + f^B)$ $-(f^A + f^B)$

Lemma 4. *Suppose that listing fees of platform A and B are identical and not zero. Also, assume that the tie breaking rule in platform selections of sellers is 0.5. Based on these assumptions, a game is defined with payoffs in Table 3. In this game, $(s_1 = A, s_2 = B)$ or $(s_1 = B, s_2 = A)$ are Nash Equilibria.*

Proof. By the assumption, $f^A = f^B \neq 0$. Also, following Lemma 3, $\eta^A(m(i), d) = 0$, $\eta^B(d, m(i)) = 0$. Then, by definition of Nash eqm, $(s_1 = A, s_2 = B)$ and $(s_1 = B, s_2 = A)$ are the equilibria. \square

2.2.5 Step 1: Platforms' listing fee setting

Lemma 5. *Suppose that sellers exist on a platform. Then, the listing fee is not larger than the total revenue of sellers under Nash Equilibrium.*

Proof. A seller predicting the minus profit can choose the outside option. \square

2.2.6 Results

Proposition 1 (Eqm 1). *The following subgame perfect Nash equilibrium exists: half of the consumers choose platform A and the other half choose B, each seller forms a monopoly on each platform, and each platform sets the listing fee equal to the seller's price.*

Proof. See the appendix.

Corollary 1. *In eqm 1, platforms extract the total economic welfare generated from transactions on the platforms.*

Table 4: Eqm 1 Payoffs

	l	A	B
u_l		$0.5 \cdot (x + \gamma \cdot 0.5)$	$0.5 \cdot (x + \gamma \cdot 0.5)$
	i	1	2
u_i		0	0
	θ	$\forall \theta$	
u_θ		0	

3 Tipping equilibrium in platform markets

3.1 Multihoming of consumers

In this section, we assume that multihoming of consumers is allowed. That is, $s_\theta \in \{A, B, MH\}$.

Proposition 2 (Eqm 2). *When multihoming of consumers is possible, there exists an equilibrium in which all sellers enter a specific platform, and multihoming consumers purchase objects from only on that platform. Under this equilibrium, the platform sets the listing fee to zero, and consumers purchase the product at a duopoly price.*

Proof. See the Appendix.

In eqm 2, consumers choose multihoming to earn bargaining power in the product market, and sellers are tipping. (Note that in this case, even if a monopoly is formed at the sellers level in each platform, price competition occurs across the platforms.) The product price is the duopoly price, which maximizes consumer surplus (Table 5).

Table 5: Eqm 2 Payoffs (consumers MH, sellers tipping on A)

	l	A	B
u_l		0	0
	i	1	2
u_i		0	0
	θ	$\forall \theta$	
u_θ		$x + \gamma \cdot 1$	

3.2 Consumers' taste for Platform A

Now we assume that consumers have a taste for platform A when the product values, prices, network effects from platform A and B are the same. That is, if all platforms form a monopoly or duopoly product market (Lemma 3(1), 3(4)), consumers choose platform A with a probability of 1. In this case, a tipping equilibrium exist as follows.

Proposition 3 (Eqm 3). *If consumers have a taste for a specific platform, there exists an equilibrium where a monopoly product market is formed and consumers are tipping on that platform.*

Proof. See the appendix.

Table 6: Eqm 3 Payoffs (taste, consumers tipping on A)

	l	A	B
u_l		$(x + \gamma \cdot 1)$	0
	i	1	2
u_i		0	0
	θ	$\forall \theta$	
u_θ		0	

3.3 Tying

Suppose that vertically integrated platform $A1$ exists s.t. platform A and seller 1 are merged. That is, platform $A1$ sets a listing fee of seller 2 and decides a price of seller 1. The profit of $A1$ is defined as follows.

$$u_{A1} = \begin{cases} f^A + \mu_1^A \cdot p_1^A, & \text{if } s_B = A1 \\ \mu_1^A \cdot p_1^A, & \text{if } [s_B \neq A1 \wedge s_B \neq MH] \end{cases}$$

Proposition 4 (Eqm 4). *If a vertically integrated platform provides platform services and sells a product(or service) simultaneously, there exists a equilibrium where consumers are tipping on the integrated platform and the platform excludes other sellers. In this case, the product is sold at a monopoly price.*

Proof. See the appendix.

Table 7: Eqm 4 Payoffs (tying, consumers tipping on A1)

	l	$A1$	B
u_l		$(x + \gamma \cdot 1)$	0
	i	N/A	2
u_i		N/A	0
	θ	$\forall \theta$	
u_θ		0	

3.4 Multi-sided Market

Now we assume that platform A also performs advertising business. That is, if platform A has an enough size of consumers (when tipping occurs), platform A can earn a large profit from the advertising market and can set the listing fee of sellers with minus values (leverage is allowed).

- $u_A = n^A \cdot f^A + 1_{\eta^A=1} \cdot \alpha$
- $f^A \in (-\infty, \infty)$
- $\alpha \geq 3 \cdot (x + \gamma \cdot 1)$

We also assume that consumers selects platform B when they are indifferent between A and B . Despite these assumptions, when the advertising within the platform is added, there is an eqm in which consumers and sellers are tipping on the platform.

Proposition 5 (Eqm 5). *When advertising services are added to a platform, there exists an eqm s.t. a) sellers and consumers are tipping on that platform, b) sellers set a duopoly price in the platform, and c) the platform provide sellers with a subsidy of a monopoly profit.*

Proof. See the appendix.

Table 8: Eqm 5 Payoffs (multi-sided market, tipping on A)

	l	$A1$	B
u_l		$\alpha - 2 \cdot (x + \gamma \cdot 1)$	0
	i	1	2
u_i		$(x + \gamma \cdot 1)$	$(x + \gamma \cdot 1)$
	θ	$\forall \theta$	
u_θ		$(x + \gamma \cdot 1)$	

4 Conclusion

(to be modified)

- existence of tipping eqm in various environments: consumers multihoming, taste, tying, multi-sided mkt.
- agents utility levels are different depending on the tipping cases.
- sellers competition is crucial for the consumers' surplus level and their strategies.
- and the tipping incentives and the process are different across the cases.
- the dominant position and the market power of the tipping platform can be strengthen throughout the process: consumers tipping => taste created => platform's tying strategy => selling Ads and subsidizing other agents

Appendix

to be added

References

- Armstrong, Mark (2006) “Competition in two-sided markets,” *The RAND journal of economics*, Vol. 37, No. 3, pp. 668–691.
- Caillaud, Bernard and Bruno Jullien (2003) “Chicken & egg: Competition among intermediation service providers,” *RAND journal of Economics*, pp. 309–328.
- Dudey, Marc (1990) “Competition by choice: The effect of consumer search on firm location decisions,” *The American Economic Review*, pp. 1092–1104.
- Ellison, Glenn and Drew Fudenberg (2003) “Knife-edge or plateau: When do market models tip?” *The Quarterly Journal of Economics*, Vol. 118, No. 4, pp. 1249–1278.
- Karle, Heiko, Martin Peitz, and Markus Reisinger (2020) “Segmentation versus agglomeration: Competition between platforms with competitive sellers,” *Journal of Political Economy*, Vol. 128, No. 6, pp. 2329–2374.
- Rochet, Jean-Charles and Jean Tirole (2003) “Platform competition in two-sided markets,” *Journal of the european economic association*, Vol. 1, No. 4, pp. 990–1029.
- (2006) “Two-sided markets: a progress report,” *The RAND journal of economics*, Vol. 37, No. 3, pp. 645–667.